

# Self-Driving Vehicle Verification Towards a Benchmark

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## ABSTRACT

Industrial cyber-physical systems are hybrid systems with strict safety requirements. Despite not having a formal semantics, most of these systems are modeled using Stateflow/Simulink<sup>®</sup> for mainly two reasons: (1) it is easier to model, test, and simulate using these tools, and (2) dynamics of these systems are not supported by most other tools. Furthermore, with the ever growing complexity of cyber-physical systems, grows the gap between what can be modeled using an automatic *formal* verification tool and models of industrial cyber-physical systems. In this paper, we present a simple formal model for self-driving cars. While after some simplification, safety of this system has already been proven manually, to the best of our knowledge, no automatic formal verification tool supports its dynamics. We hope this serves as a challenge problem for formal verification tools targeting industrial applications.

## KEYWORDS

Model Checking, Cyber-Physical System, Challenge Problem, Automatic Formal Verification

## 1 INTRODUCTION

The following paragraph is taken directly from [25]. According to the authors, two ingredients are missing from the race for who will have the first self-driving car on the road: (1) standardization of safety assurance, and (2) scalability.

The “Winter of AI” is commonly known as the decades long period of inactivity following the collapse of Artificial Intelligence research that over-reached its goals and hyped its promise until the inevitable fall during the early 80s. We believe that the development of Autonomous Vehicles (AV) is dangerously moving along a similar path that might end in great disappointment after which further progress will come to a halt for many years to come.

A typical approach to estimate the amount of safety assurance while preserving scalability, is to use statistical techniques, in which one simulates the system or collects actual/random data. To appreciate the problematic nature of a data-driven approach, authors in [25] prove, in order to have  $10^{-9}$  fatality per hour in an autonomous vehicle, one require  $10^9$  hours of data, which for example means, 1000 employees must drive 24 hours a day, 7 days a week, for 114 years! Even worse, every time part of a system gets updated,

no matter how small, preserving the guarantee requires repeating the whole data collection.

To solve the safety standardization problem, [25] suggests the notion of “who is responsible” for an accident in a non-deterministic setting. Intuitively, instead of trying to build a system in which no accident occurs, “Responsibility-Sensitive Safety”, tries to prevent car  $c$  from only those accidents in which  $c$  is going to be blamed. In other words, if a car drives responsibly, it might still be involved in an accident, but it will never be blamed for one. To achieve this goal, [25] defines two major components: (1) a policy that cars should follow, and (2) a mechanism to identify responsible parity (or parties) in case of an accident (they are exactly those who will be blamed for the accident). The majority of the paper is devoted to different policies in different conditions, like moving in the same direction or in the opposite directions, moving laterally or longitudinally, moving on a straight road or road with other geometries, and who should be blamed in case of accident in each of those conditions.

After a policy is defined, one has to show it does not blame those who follow it and at least one party in each accident will be blamed (otherwise a policy that prevents nothing won’t blame anyone for an accident, but also does not prevent any accident from happening). Unfortunately, all these proofs or in some cases only sketches of proofs are done manually in [25]. However, to the best of our knowledge, there is no automatic formal verification tool that can be used to prove these properties. Even worse, we are not aware of any automatic formal verification tool that can be used to specify these properties. This was our motivation to write this paper, in which we specify the most basic and fundamental policies defined in [25] and challenge current and any future automatic formal verification tool for cyber-physical system to solve any of the five challenge problems in this paper.

In Section 2, we review preliminary definitions we need in this paper. In Section 3, we formally specify system and policy defined in [25], for the case when finite number of cars are driving on a straight road. The rigorous level of the specifications in this paper is high enough to seamlessly write them all in a theorem prover like PVS [18] or Lean [5]. This removes any ambiguities from policy and system dynamics<sup>1</sup>. Having policy and system dynamics clearly defined, next we specify five different fundamental problems about these specification in Section 4. The first four are about (robust

<sup>1</sup>Through this process we observed a couple of problems/inconsistencies with the specifications in [25] (they are mentioned at different places in this paper).

safety and (robust) liveness, and the last one, is about the policy when not every car follows it. We use signals (a function from a non-negative real value as time to a point in a metric space) to specify all of our system dynamics, policy, and problems. This makes our specifications uniform but not constructive, *i.e.* it does not specify how to build a system that follows those specifications. In Section 5, we list nine different tools and six different reasons that prevent us from even specifying our problems in these tools. This is after ignoring all the difficulties that may arise when one wants to encode everything in the language of one of these tools. All these tools are written solely for the purpose of formal model checking cyber-physical systems. Finally, we conclude the paper in Section 6.

## 2 PRELIMINARIES

We denote the set of *natural*, *positive natural*, *real*, *positive real*, and *non-negative real* numbers by  $\mathbb{N}$ ,  $\mathbb{N}_+$ ,  $\mathbb{R}$ ,  $\mathbb{R}_+$ , and  $\mathbb{R}_{\geq 0}$ , respectively. For any two sets  $A$  and  $B$ , *size* of  $A$  is denoted by  $|A|$ , and *the set of functions from  $A$  to  $B$*  is denoted by  $A \rightarrow B$  or  $B^A$ . Operator  $\rightarrow$  is considered to be right-associative, meaning if  $C$  is a set then function  $f$  of type  $A \rightarrow B \rightarrow C$  is a function that maps every element of type  $A$  to an element of type  $B \rightarrow C$ .

### 2.1 Extended Metric Space and Distance Functions

Let  $M$  be an arbitrary set and  $d : M \times M \rightarrow \mathbb{R} \cup \{\infty\}$  be an arbitrary function. Ordered pair  $(M, d)$  is called an *extended metric space* and  $d$  is called a *distance function* iff for any  $x, y, z : M$  the following conditions hold: (1)  $d(x, y) \geq 0$ , (2)  $d(x, y) = 0 \Leftrightarrow x = y$ , (3)  $d(x, y) = d(y, x)$ , and (4)  $d(x, z) \leq d(x, y) + d(y, z)$ . If distance function  $d$  is known from the context, we use  $M$  to also refer to the metric space. Let  $X$  be a finite set of variables, and  $M \subseteq \mathbb{R}^X$  be an arbitrary set. A well-known distance function on  $M$ , denoted by  $d_\infty(v_1, v_2)$ , maps any two points  $v_1, v_2 : M$  to  $\max_{x \in X} |v_1(x) - v_2(x)|$ . Let  $C$  be a finite set and  $(M, d)$  be a metric space. We extend  $d$  to map any two points  $v_1, v_2 : C \rightarrow M$  to  $\max_{c \in C} d(v_1(c), v_2(c))$ .

### 2.2 Signal

In this paper we present dynamics of a system and policies both using signals. This is for two reasons: First, using one formalism to specify both behavior and policy makes our presentation more uniform. Second, we intentionally stay away from any class of hybrid automata or temporal logic, and leave it to the reader to choose or develop an appropriate formalism for this benchmark.

**DEFINITION 1 (SIGNAL).** *Let  $(M, d)$  be an extended metric space. Signal is any function of type  $\mathbb{R}_{\geq 0} \rightarrow M$ .*

Signal  $f$  is *continuous at time  $t : \mathbb{R}_{\geq 0}$*  iff  $\lim_{t' \rightarrow t} f(t')$  is defined and equal to  $f(t)$ <sup>2</sup>. Signal  $f$  is *continuous* iff it is continuous at all times. Signal  $f$  is *piecewise continuous* iff number of discontinuities within any finite amount of time is finite.

<sup>2</sup>When  $t = 0$ , we only consider continuity from right.

## 3 DYNAMICS AND POLICY

In this paper we only consider the case in which cars are on a straight road and no car drives in reverse gear. While making the presentation simpler, this is quite enough to specify our automatic formal verification challenge problems. Table 1 lists every parameter that we use in this paper.

Parameter	Description
$C$	Finite Set of Cars
$\rho : \mathbb{R}_{\geq 0}$	Response Time
$\mu : \mathbb{R}_+$	Minimum Distance Parameter
$a_{\max\_accel} : \mathbb{R}_+$	Maximum Longitudinal Acceleration
$a_{\min\_brake} : \mathbb{R}_+$	Minimum Longitudinal Deceleration
$a_{\max\_brake} : \mathbb{R}_+$	Maximum Longitudinal Deceleration
$a_{\max\_accel}^{\text{lat}} : \mathbb{R}_+$	Maximum Lateral Acceleration
$a_{\min\_brake}^{\text{lat}} : \mathbb{R}_+$	Minimum Lateral Deceleration
$a_{\max\_brake}^{\text{lat}} : \mathbb{R}_+$	Maximum Lateral Deceleration

**Table 1: Parameters**

The most basic signal in this paper is the position signal that specifies position of every car on the road throughout the entire time. The next most fundamental signal in this paper is the delay signal that models delays in the controller parts of a cyber-physical system. We first define position and delay signals. Next, we define minimum longitudinal and lateral distance signals as a function of position and delay signals. Later, we use these four signals to uniquely define dangerous situation and blame time signals. These signals together are almost everything we need to define policy and verification problems about that policy.

**DEFINITION 2 (POSITION SIGNAL).** *Let  $C$  be an arbitrary finite set of cars. Position signal is any function of the type  $f : \mathbb{R}_{\geq 0} \rightarrow C \rightarrow \mathbb{R}^{\{x, y\}}$ .*

Note that in Definition 2,  $C \rightarrow \mathbb{R}^{\{x, y\}}$  is the metric space. Also, in Definition 2 and every other signal that is defined later in this paper, we use  $d_\infty$  as the distance function. Let  $f$  be a position signal. We say  $f$  is *differentiable at time  $t : \mathbb{R}_{\geq 0}$*  iff for every car  $c : C$  and axis  $u : \{x, y\}$ ,  $\lim_{h \rightarrow 0} \frac{(f(t+h) c u) - (f t c u)}{h}$  is defined<sup>3</sup>. We say  $f$  is *differentiable* iff it is differentiable at all times in  $\mathbb{R}_{\geq 0}$ . Furthermore, we use  $f_v$  to denote derivative of  $f$  and call it *velocity signal* (note that  $f_v$  is also a position signal). Furthermore, if  $f_v$  is differentiable, we denote the induced derivative signal by  $f_a$  and call it *acceleration signal*. Finally, we use  $\text{Pos}_C$  to denote the set of position signals  $f$  with two conditions: (1) both  $f_v$  and  $f_a$  are defined throughout the entire time domain, and (2) no car has a negative longitudinal velocity (*i.e.*  $\forall t : \mathbb{R}_{\geq 0}, c : C \bullet f_v t c y \geq 0$ ).

Position of different cars is a physical property of our cyber-physical system and for every signal, each car has a unique position at every single point in time. However, when a car uses its sensors to observe positions of different cars including itself, there are at least

<sup>3</sup>Similar to the continuity definition, if  $t = 0$ , we only consider differentiability from right.

two sources of errors: (1) *measurement errors* caused by inaccuracy of sensors, and (2) *slight delay* in sensors and controllers (parameter  $\rho$  in Table 1). To handle measurement errors, one has to consider not only a position signal, but *at least* all the position signals that are pointwise close to it. This usually happens in the context of *robust verification* [7, 9, 10, 13, 17, 26, 27]. Delays on the other hand, are usually considered in system models [1, 2, 6, 19, 23], which is the focus of this section. In order to simplify presentation of later definitions, we next define a delay signal that assigns a delay to every pair of cars. If  $\tau$  is a delay signal, its value for cars  $c_1, c_2 : C$  at time  $t : \mathbb{R}_{\geq 0}$  models a delayed time in car  $c_1$  when it observes state of car  $c_2$  at time  $t$ . Although, we use one delay signal throughout our entire formulation, one can easily extend this to multiple delay signals, one for each part of the system. Definition 3 formally defines a delay signal based on response time parameter  $\rho$ . Note that, by definition, there is no delay at time  $t = 0$ . Also, if  $\rho = 0$  then there will be no delay in the future either.

**DEFINITION 3 (DELAY SIGNAL).** Delay signal is any piecewise continuous signal of type  $\mathbb{R}_{\geq 0} \rightarrow C \rightarrow C \rightarrow \mathbb{R}_{\geq 0}$  that satisfies  $\forall t : \mathbb{R}_{\geq 0}, c_1, c_2 : C \cdot t - \rho \leq \tau t c_1 c_2 \leq t$ , where  $\rho : \mathbb{R}_{\geq 0}$  is defined in Table 1. We use  $\text{Delay}_C$  to denote the set of all delay signals for cars in  $C$ .

The general idea in [25] to guarantee safety is to first define a safe distance between every two cars and then take a proper action whenever distance is unsafe. The safe distance is computed using the knowledge a car has about velocity of itself and another car, and is *supposed to be* large enough such that the car will have enough time to respond properly, whenever the distance becomes unsafe. Definition 4 and Definition 5 define minimum (safe) longitudinal and lateral distances, respectively.

**DEFINITION 4 (MINIMUM LONGITUDINAL DISTANCE SIGNAL).** Let  $f$  and  $\tau$  be, respectively, position and delay signals. We define minimum longitudinal distance signal, denoted by  $d_{\min}^{\text{lon}}$ , as a function of type  $\mathbb{R}_{\geq 0} \rightarrow C \rightarrow C \rightarrow \mathbb{R}_+ \cup \{-\infty\}$  that maps a time  $t$  and cars  $c_1, c_2 : C$  with  $t_1 := \tau t c_1 c_1$  and  $t_2 := \tau t c_1 c_2$  to  $-\infty$  if  $c_1 = c_2$  or  $f t_1 c_1 y > f t_2 c_2 y$ , and to the maximum of  $\mu$  and following term, otherwise:

$$\rho f_v t_1 c_1 y + \frac{1}{2} a_{\max\_accel} \rho^2 + \frac{(f_v t_1 c_1 y + \rho a_{\max\_accel})^2}{2a_{\min\_brake}} - \frac{(f_v t_2 c_2 y)^2}{2a_{\max\_brake}}$$

There is a big difference between Definition 4 and its correspondence in [25]. Definition 4 uses delayed observations, but minimum distance in [25] is defined assuming exact value of every car's longitudinal velocity is known to every other car at all times. Another difference is that, in Definition 4, we make sure minimum distance is never smaller than  $\mu$ , however in [25] this distance can become arbitrary close to 0. Since both Definition 5 below and [25] make sure that minimum lateral distance is never smaller than  $\mu$ , our approach is more uniform.

**DEFINITION 5 (MINIMUM LATERAL DISTANCE SIGNAL).** Let  $f$  and  $\tau$  be, respectively, position and delay signals. We define minimum lateral distance signal, denoted by  $d_{\min}^{\text{lat}}$ , as a function of type  $\mathbb{R}_{\geq 0} \rightarrow C \rightarrow C \rightarrow \mathbb{R}_+ \cup \{-\infty\}$  that maps a time  $t$  and cars  $c_1, c_2 : C$  with

$t_1 := \tau t c_1 c_1$  and  $t_2 := \tau t c_1 c_2$  to  $-\infty$  if  $c_1 = c_2$ , and to the following term, otherwise:

$$\mu + \rho |f_v t_1 c_1 x| + \rho |f_v t_2 c_2 x| + \frac{1}{2} a_{\max\_accel}^{\text{lat}} \rho^2 + \frac{(|f_v t_1 c_1 x| + \rho a_{\max\_accel}^{\text{lat}})^2}{2a_{\min\_brake}^{\text{lat}}} + \frac{1}{2} a_{\max\_accel}^{\text{lat}} \rho^2 + \frac{(|f_v t_2 c_2 x| + \rho a_{\max\_accel}^{\text{lat}})^2}{2a_{\min\_brake}^{\text{lat}}}$$

Note that similar to Definition 4, minimum distance in Definition 5 is also computed using delayed observations. The only other difference between Definition 5 and its correspondence in [25] is that we do not assume car  $c_1$  is on the left of car  $c_2$ . Finally, to the best of our knowledge, the case when two cars move laterally in the same direction is not considered in [25] and hence nor here <sup>4</sup>.

According to Definition 2 and what comes after it, for any position signal  $f$ , there are unique velocity and acceleration signals. However, according to Definition 4 and Definition 5, when response time ( $\rho$ ) is positive, there could be uncountably many minimum longitudinal/lateral distance signals for  $f$ . This is because, we assume velocity and acceleration are physical properties that are defined using position. For example, if positions at times 1 and 3 are respectively 10 and 18 then (average) velocity during this time is exactly  $\frac{18-10}{3-1}$ . However, we assume actual values of these signals are obtained/observed with delay of at most  $\rho$  units of time. Signal  $f$  (and hence signals  $f_v$  and  $f_a$ ) can take uncountably many values during any positive duration of time. Therefore, there are uncountably many possible minimum longitudinal/lateral distance signals that can be observed/considered.

**DEFINITION 6 (DANGEROUS LONGITUDINAL SITUATION SIGNAL).** Let  $f$ ,  $\tau$ , and  $d_{\min}^{\text{lon}}$  be a position, delay, and minimum longitudinal distance signals, respectively. We define dangerous longitudinal situation signal, denoted by  $\text{dang}^{\text{lon}}$ , as a function of type  $\mathbb{R}_{\geq 0} \rightarrow C \rightarrow C \rightarrow \{\top, \perp\}$  that maps a time  $t$  and cars  $c_1, c_2 : C$  with  $t_1 := \tau t c_1 c_1$  and  $t_2 := \tau t c_1 c_2$  to  $\top$  exactly when value of  $d_{\min}^{\text{lon}} t c_1 c_2$  is strictly larger than  $(f t_2 c_2 y) - (f t_1 c_1 y)$ .

**DEFINITION 7 (DANGEROUS LATERAL SITUATION SIGNAL).** Let  $f$ ,  $\tau$ , and  $d_{\min}^{\text{lat}}$  be a position, delay, and minimum lateral distance signals, respectively. We define dangerous lateral situation signal, denoted by  $\text{dang}^{\text{lat}}$ , as a function of type  $\mathbb{R}_{\geq 0} \rightarrow C \rightarrow C \rightarrow \{\top, \perp\}$  that maps a time  $t$  and cars  $c_1, c_2 : C$  with  $t_1 := \tau t c_1 c_1$  and  $t_2 := \tau t c_1 c_2$  to  $\top$  exactly when value of  $d_{\min}^{\text{lat}} t c_1 c_2$  is strictly larger than  $|(f t_2 c_2 x) - (f t_1 c_1 x)|$ .

We define *dangerous situation signal*, denoted by  $\text{dang}$ , as a function of type  $\mathbb{R}_{\geq 0} \rightarrow C \rightarrow C \rightarrow \{\top, \perp\}$  that maps time  $t : \mathbb{R}_{\geq 0}$  and cars  $c_1, c_2 : C$  to the conjunction of  $(\text{dang}^{\text{lon}} t c_1 c_2)$  and  $(\text{dang}^{\text{lat}} t c_1 c_2)$ . Note that Definition 6 and Definition 7 also use delayed observations in their definitions.

**DEFINITION 8 (BLAME TIME SIGNAL).** Let  $\text{dang}$  be a dangerous situation signal. We define blame time signal, denoted by  $\text{blame}$ , as a function of type  $\mathbb{R}_{\geq 0} \rightarrow C \rightarrow C \rightarrow \mathbb{R}_{\geq 0} \cup \{\infty\}$  that maps time  $t : \mathbb{R}_{\geq 0}$  and cars  $c_1, c_2 : C$  to

<sup>4</sup>We leave it to the reader to prove or disprove the necessity of considering that case.

$$\begin{aligned} \infty & \text{ if } \neg \text{dang } t \ c_1 \ c_2 \text{ or } \forall r : [0, t] \bullet \text{dang } t \ c_1 \ c_2^5 \\ t' & \text{ if } \forall r : (t', t] \bullet \text{dang } r \ c_1 \ c_2 \text{ and} \\ & \forall t'' : [0, t') \bullet \exists r' : (t'', t') \bullet \neg \text{dang } r' \ c_1 \ c_2 \end{aligned}$$

We denote the blame time signals that are obtained by replacing *dang* with *dang<sup>lon</sup>* and *dang<sup>lat</sup>*, respectively by *blame<sup>lon</sup>* and *blame<sup>lat</sup>*.

The second condition in Definition 8 uniquely defines value of  $t'$ . Intuitively, it is the smallest value  $t'$  for which the situation is dangerous at any time between  $t'$  and  $t$ . Note that according to [25], in the second case of Definition 8, instead of  $\forall t'' : [0, t') \bullet \exists r' : (t'', t') \bullet \neg \text{dang } r' \ c_1 \ c_2$ , we should have just said  $\neg \text{dang } t' \ c_1 \ c_2$ . It is easy to see that our condition is strictly weaker. For example, if  $t' > 0$ ,  $\text{dang } t' \ c_1 \ c_2 = \top$  and  $\forall r' : (0, t') \bullet \neg \text{dang } r' \ c_1 \ c_2$  then value of *blame*  $t \ c_1 \ c_2$ , according to Definition 8 is  $t'$ , and according to [25] is undefined. We leave it to the reader to (dis)prove that Definition 6 and Definition 7 and whatever comes before them guarantee dangerous signal is continuous from left, in which case Definition 8 and its correspondent in [25] are equivalent.

According to the following quote from [25], by simply not moving, a car can have 0 longitudinal velocity for a long time, but it is impossible for a car to keep its lateral velocity at 0. We believe this is a mistake, since if a car does not move then it has zero velocity in both directions. Furthermore, since we only consider position signals with fully defined velocity and acceleration, the velocity signal is continuous throughout the entire time. Therefore, whenever its sign is different at time  $t_1$  and  $t_2$ , we know its value is 0 at some time between  $t_1$  and  $t_2$ , which is enough for the purpose of this paper.

Unlike longitudinal velocity, which can be kept to a value of 0 for a long time (the car is simply not moving), keeping lateral velocity at exact 0 is impossible as cars usually perform small lateral fluctuations. It is therefore required to introduce a robust notion of lateral velocity.

We have everything we need to finally define a policy in Definition 9.

**DEFINITION 9 (POLICY).** *Let  $f$  and  $\tau$  be position and delay signals, respectively, and let signals  $d_{\min}^{\text{lon}}$ ,  $d_{\min}^{\text{lat}}$ ,  $\text{dang}^{\text{lon}}$ ,  $\text{dang}^{\text{lat}}$ ,  $\text{dang}$ ,  $\text{blame}^{\text{lon}}$ ,  $\text{blame}^{\text{lat}}$ , and  $\text{blame}$  be uniquely defined based on  $f$  and  $\tau$ , as specified in this section. For any time  $t : \mathbb{R}_{\geq 0}$  and car  $c_1 : C$ , we say car  $c_1$  follows the policy at time  $t$ , denoted by  $\mathbb{P} f \ \tau \ t \ c_1$  iff for any car  $c_2 : C$ , if  $\text{dang } t \ c_1 \ c_2 = \top$  and  $t_b := \text{blame } t \ c_1 \ c_2 \in \mathbb{R}$  then the following conditions hold:*

- *If before the blame time there was a safe longitudinal distance between  $c_1$  and  $c_2$  (i.e.  $t_b = \text{blame}^{\text{lon}} \ t \ c_1 \ c_2$ ) then*
  - (1)  $\forall t' : (t_b, t_b + \rho) \bullet f_a \ t' \ c_1 \ y \leq a_{\max\_accel}$ , i.e. *within the response time, acceleration of the rear car must be bounded by  $a_{\max\_accel}$ .*
  - (2)  $\forall t' : [t_b + \rho, t] \bullet f_a \ t' \ c_1 \ y \leq -a_{\min\_brake}$ , i.e. *after the response time, bound on acceleration decreases to  $-a_{\min\_brake}$  (the rear car must use its brake).*

<sup>5</sup>The case  $\forall r : [0, t] \bullet \text{dang } t \ c_1 \ c_2$  is only considered here for completeness. However, using additional constraints that will be given later, we would not consider any signal that is initially dangerous, i.e. satisfies  $\exists c_1, c_2 : C \bullet \text{dang } 0 \ c_1 \ c_2$  (note that value of  $\text{dang } 0 \ c_1 \ c_2$  is uniquely determined by values of  $f \ 0 \ c_1 \ c_2$  and  $f_v \ 0 \ c_1 \ c_2$ ).

(3)  $\forall t' : (t_b, t] \bullet f_a \ t' \ c_2 \ y \geq -a_{\max\_brake}$ , i.e. *there is bound on how fast the front car can stop.*

- *If before the blame time there was a safe lateral distance between  $c_1$  and  $c_2$  (i.e.  $t_b = \text{blame}^{\text{lat}} \ t \ c_1 \ c_2$ ) then*
  - (1)  $\forall t' : (t_b, t_b + \rho) \bullet |f_a \ t' \ c_1 \ x| \leq a_{\max\_accel}^{\text{lat}}$ , i.e. *within the response time, acceleration of car  $c_1$  must be bounded by  $a_{\max\_accel}^{\text{lat}}$ .*
  - (2)  $\forall t' : [t_b + \rho, t] \bullet |f_a \ t' \ c_1 \ x| \leq a_{\min\_brake}^{\text{lat}}$  and  $(f_v \ t' \ c_1 \ x) \times (f_a \ t' \ c_1 \ x) \leq 0$ , i.e. *after the response time, bound on acceleration decreases to  $a_{\min\_brake}^{\text{lat}}$  and acceleration and velocities are in the opposite direction. (a.k.a. the  $c_1$  must use its brake).*

We define  $\mathbb{P} f \ \tau \ c_1$  to be  $\forall t : \mathbb{R}_{\geq 0} \bullet \mathbb{P} f \ \tau \ t \ c_1$  (i.e. *car  $c_1$  always follows the policy*). Similarly, we define  $\mathbb{P} f \ \tau$  to be  $\forall c_1 : C \bullet \mathbb{P} f \ \tau \ c_1$  (i.e. *every car follows the policy at all time*).

There are three differences between the first part of policy written in Definition 9 and the one introduced in [25]. First, according to Definition 9, there is no requirement on acceleration of the rear car at time  $t_b$ . We believe imposing a restriction at time  $t_b$  is a mistake, specially in [25], since by definition the situation is not dangerous at  $t_b$  and no car can look into the future of the system state. The next two differences are more important. According to [25], after the rear car reached to full stop, it can never move forward. Similarly, after the front car reached to full stop it can never decelerate. We believe either one of these policies is too restrictive to be allowed in any real scenario. One implies if the rear car enters a dangerous situation, it is going to stop on the road and never move again. The other one implies if the front car enter into a dangerous situation with a car on its behind, first it will fully stop and then if it moves, it will never lower its speed. None of these makes any sense in practical scenarios. These three differences also exists between the second part of policy written in Definition 9 and the one introduced in [25].

## 4 VERIFICATION PROBLEMS

We have specified dynamics and policy of cars in Section 3. In this section we specify multiple verification problems about those specifications. According to Section 3, for every position and delay signals, minimum longitudinal and lateral distance signals ( $d_{\min}^{\text{lon}}$  and  $d_{\min}^{\text{lat}}$ ), longitudinal and lateral dangerous situation signals ( $\text{dang}^{\text{lon}}$ ,  $\text{dang}^{\text{lat}}$ , and  $\text{dang}$ ), and longitudinal and lateral blame time signals ( $\text{blame}^{\text{lon}}$ ,  $\text{blame}^{\text{lat}}$ , and  $\text{blame}$ ), are all uniquely defined. Therefore, in this section, whenever we consider a position and a delay signal, we assume all the other signals can be used without introduction. We divide our verification problems into three different categories: (1) safety properties, (2) liveness properties, and (3) responsibility properties.

### 4.1 Safety Problems

**PROBLEM 10 (SAFETY).** *Prove or disprove that policy in Definition 9 guarantees utopia (i.e. prevents accident). More precisely, prove or disprove the following formula cannot be satisfied by a position signal*

$f : \text{Loc}_C$ :

$$\underbrace{\left( \exists \tau : \text{Delay}_C \bullet \underbrace{\left( \forall c_1, c_2 : C \bullet \neg \text{dang } 0 \ c_1 \ c_2 \right)}_{\mathcal{A}_2} \wedge \mathbb{P} f \ \tau \right)}_{\mathcal{A}_1} \wedge \underbrace{\left( \exists t : \mathbb{R}_{>0}, c_1, c_2 : C \bullet f \ t \ c_1 = f \ t \ c_2 \right)}_{\mathcal{A}_3}$$

Condition  $\mathcal{A}_2$  guarantees that the situation is not initially dangerous. Condition  $\mathcal{A}_1$  guarantees  $f$  is initially not dangerous and it follows policy as specified in Definition 9. Condition  $\mathcal{A}_3$  guarantees that there will be an accident in the future. A system/policy is *safe* iff the formula defined in Problem 10 is unsatisfiable. Finally note that formula defined in Problem 10 depends on parameters given in Table 1. We leave it to the reader to solve this problem for only one or a class of values of these parameters.

Problem 10 ultimately depends on signals  $d_{\min}^{\text{lat}}$  and  $d_{\min}^{\text{lon}}$ . Distances defined by these two signals are never smaller than the same distances defined in [25]. However, as we mentioned multiple times, there is a big difference here: in this paper definitions of signals  $d_{\min}^{\text{lat}}$  and  $d_{\min}^{\text{lon}}$  involve delay, while in [25] these signals are defined using no delay (*i.e.* response time is zero)<sup>6</sup>. Furthermore, minimum/safe distance defined in Definition 4 uses the fact that observations are made with no delay. This intuitively means that, using policy and minimum distance defined in [25], cars can become arbitrary close to each other. Therefore, it should be of no surprise that if we compute minimum distance the same way as in [25], but use delayed values for it, cars will crash. This informal justification answers Problem 10 negatively. However, it is not clear to us how one should fix this problem. For example, if we consider delay, is it still true that the minimum distance is always exists, or even to guarantee its existence one has to bound both velocity and acceleration (policy in Definition 9 only bounds signals during some intervals)? Furthermore, validity of any suggestion for fixing this issue requires a formal proof, something that we look forward to be done automatically.

Problem 10 completely ignores errors and uncertainties in each cars' sensors. As mentioned before, this is usually handled in the context of robust verification. Note that there are many definitions for robustness. What we put here is taken from [13] and is for illustration purposes only. For any position signal  $f : \text{Loc}_C$  and  $\epsilon : \mathbb{R}_{>0}$ , let  $B_\infty^\epsilon(f)$  be the set of signals in  $\text{Loc}_C$  that are point-wise  $\epsilon$ -close to  $f$ . More precisely,  $f' \in B_\infty^\epsilon(f)$  iff  $\sup_{t:\mathbb{R}_{>0}} d(f \ t, f' \ t) \leq \epsilon$ , where  $d$  is the distance function used in the definition of signal.

**DEFINITION 11 ( $\epsilon$ -ROBUST SAFE AND UNSAFE SIGNALS).** *Let  $\mathcal{A}_1$  and  $\mathcal{A}_3$  be the two predicates over position signals defined in Problem 10. A position signal  $f : \text{Loc}_C$  is called  $\epsilon$ -robust safe iff it satisfies the following formula:*

$$\mathcal{A}_1(f) \Rightarrow \forall f' : B_\infty^\epsilon(f) \bullet \neg \mathcal{A}_3(f')$$

<sup>6</sup>Authors in [25] only consider positive response time when a car responds to a dangerous situation. However, observing position and velocity of the every other car that is used to determine if a situation is dangerous is assumed to be performed within 0 response time.

Similarly,  $f : \text{Loc}_C$  is called  $\epsilon$ -robust unsafe iff it satisfies the following formula:

$$\mathcal{A}_1(f) \wedge \forall f' : B_\infty^\epsilon(f) \bullet \mathcal{A}_3(f')$$

A position signal is called *robustly safe (unsafe)* iff it is  $\epsilon$ -robust safe (unsafe) for some  $\epsilon : \mathbb{R}_+$ . A policy is called  $\epsilon$ -robust safe (unsafe) iff all (some) position signals are  $\epsilon$ -robust safe (unsafe) in that policy. A policy is called *robustly safe (unsafe)* iff all (some) position signals are *robustly safe (unsafe)* in that policy.

Note that it is impossible for a position signal (or a policy) to be both robustly safe and unsafe. But it is possible for a position signal (or a policy) to be neither robustly safe nor robustly unsafe.

**PROBLEM 12 (ROBUST SAFETY).** *Prove or disprove that the policy in Definition 9 is robustly safe (or robustly unsafe). More precisely, determine which of the following sentences are true and which ones are false:*

- (1)  $\epsilon$ -robust safe:  $\forall f : \text{Loc}_C \bullet \mathcal{A}_1(f) \Rightarrow \forall f' : B_\infty^\epsilon(f) \bullet \neg \mathcal{A}_3(f')$
- (2)  $\epsilon$ -robust unsafe:  $\exists f : \text{Loc}_C \bullet \mathcal{A}_1(f) \wedge \forall f' : B_\infty^\epsilon(f) \bullet \mathcal{A}_3(f')$
- (3) *robustly safe*:  
 $\forall f : \text{Loc}_C \bullet \exists \epsilon : \mathbb{R}_+ \bullet \mathcal{A}_1(f) \Rightarrow \forall f' : B_\infty^\epsilon(f) \bullet \neg \mathcal{A}_3(f')$
- (4) *robustly unsafe*:  
 $\exists f : \text{Loc}_C \bullet \exists \epsilon : \mathbb{R}_+ \bullet \mathcal{A}_1(f) \wedge \forall f' : B_\infty^\epsilon(f) \bullet \mathcal{A}_3(f')$

It should be easy to see that being  $\epsilon$ -robust safe (unsafe) implies being robustly safe (unsafe). But the converse is not necessarily true.

## 4.2 Liveness Problems

Having a safe system is not enough, otherwise one could write false as the simplest policy that guarantees safety of every system (*i.e.* any behavior that satisfies this policy is safe). We also need to make sure that it is possible for a signal to satisfy the policy that is specified in Definition 9.

**PROBLEM 13 (LIVENESS).** *Prove or disprove that policy in Definition 9 is not inconsistent (*i.e.* it can be followed). More precisely, prove or disprove the following formula can be satisfied by a position signal  $f : \text{Loc}_C$ :*

$$\forall \tau : \text{Delay}_C \bullet (\forall c_1, c_2 : C \bullet \neg \text{dang } 0 \ c_1 \ c_2) \wedge \mathbb{P} f \ \tau$$

In Problem 10 we have  $\exists \tau : \text{Delay}_C$ , but in Problem 13 we have  $\forall \tau : \text{Delay}_C$ . We chose to have this change, since for safety we want to say using any valid delay signal that together with the position signal follow the policy, results in a safe behavior. However, for liveness we want to say there is a position signal for which *any* delay signal can be used to follow the policy. This is because the intended use of delay is to allow a bounded amount of response time, and any amount of delay within this bound should be allowed by the policy.

Similar to the case of Problem 10 vs. Problem 13, just having liveness is not enough. Otherwise, although in theory there is a signal  $f : \text{Loc}_C$  that follows the policy, in practice, a car has to always behave exactly like  $f$  (any deviation violates the policy) which is never possible.

**DEFINITION 14 ( $\epsilon$ -ROBUST LIVE SIGNALS).** *A position signal  $f : \text{Loc}_C$  is called  $\epsilon$ -robust live iff it satisfies the following formula:*

$$\forall f' : B_\infty^\epsilon(f) \bullet f' \text{ satisfies the formula defined in Problem 13}$$

A position signal is called *robustly live* iff it is  $\epsilon$ -robust live for some  $\epsilon : \mathbb{R}_+$ . A policy is called *robustly live* iff some position signal  $f$  in  $\text{Pos}_C$  is *robustly live* in it.

**PROBLEM 15 (ROBUST LIVENESS).** *Prove or disprove that the policy in Definition 9 is robustly live.*

Our definition of (robust) liveness is the minimum requirement for system to be considered live and in practice one has to add more constraints to it. For example, in order to consider a position signal  $f$  live, one might want to also consider the following two constraints. (1) Longitudinal position of every car diverges to infinity ( $\forall u : \mathbb{R}, c : C \bullet \exists t : \mathbb{R}_{\geq 0} \bullet f \ t \ c \ y > u$ ). Otherwise, a policy that does not move any car will be considered  $\infty$ -robust safe and live. (2) There are always points in time at which all cars are moving for a positive duration of time ( $\forall t : \mathbb{R}_{\geq 0} \bullet \exists t_1 : (t, \infty), t_2 : (t_1, \infty) \bullet \forall r : (t_1, t_2), c : C \bullet f_v \ t \ c \ y > 0$ ). Otherwise, a policy that moves only one car at a time can be considered robustly live. *Determining* the exact set of constraints for liveness is *not* a formal process and should be determined using experience or simulation.

### 4.3 Responsibility-Sensitive Safety Problem

Our problems in Section 4.1 and Section 4.2 only concern the case in which every car follows the policy. However, there is always someone on the road who drives recklessly. Authors in [25], introduce the concept of “who is responsible for an accident”, and instead of trying to come up with a policy that guarantees absence of an accident, they come of with a policy that guarantees if a car follows the policy then it won’t be held responsible for an accident.

**DEFINITION 16 (RESPONSIBILITY FOR AN ACCIDENT).** *Let  $f$  and  $\tau$  be position and delay signals, respectively. Let  $c_1, c_2 : C$  be two cars, and let  $t : \mathbb{R}_{\geq 0}$  be a time of accident between  $c_1$  and  $c_2$  (i.e.  $f \ t \ c_1 = f \ t \ c_2$ ). We say  $c_1$  is responsible for the accident with  $c_2$  at time  $t$  iff  $\text{dang } t \ c_1 \ c_2 = \top$  and  $c_1$  did not follow the policy (as specified in Definition 9) at sometime during  $(t_b, t]$ , where  $t_b := \text{blame } t \ c_1 \ c_2$ .*

Once again considering delays distinguishes Definition 16 from the same definition in [25]. For example, because of delays, blame time ( $t_b$ ) for an accident could be different in  $c_1$  and  $c_2$ . Even worse, it is not so much obvious that whenever there is an accident, there will be a blame time. We consider these problems next. However, it should be obvious that according to Definition 16, whoever follows the policy won’t be held responsible for an accident.

**THEOREM 17 (RESPONSIBILITY-SENSITIVE SAFETY).** *Whoever follows the policy won’t be held responsible for an accident.*

**PROBLEM 18 (EXISTENCE OF RESPONSIBLE PARTY).** *Prove or disprove that each accident has at least one responsible party. More precisely, prove the following formula cannot be satisfied by any signal  $f : \text{Loc}_C$ .*

$$\begin{aligned} & \exists \tau : \text{Delay}_C, c_1, c_2 : C, t : \mathbb{R}_{\geq 0}, t_{b_1}, t_{b_2} : \mathbb{R}_{\geq 0} \cup \{\infty\} \bullet \\ & f \ t \ c_1 = c \ t \ c_2 \wedge \neg \text{dang } 0 \ c_1 \ c_2 \wedge \neg \text{dang } 0 \ c_2 \ c_1 \\ & \left( t_{b_1} = \text{blame } t \ c_1 \ c_2 \wedge \forall t' : (t_{b_1}, t] \cap \mathbb{R}_{\geq 0} \bullet \mathbb{P} f \ t' \ \tau \ c_1 \right) \wedge \\ & \left( t_{b_2} = \text{blame } t \ c_2 \ c_1 \wedge \forall t' : (t_{b_2}, t] \cap \mathbb{R}_{\geq 0} \bullet \mathbb{P} f \ t' \ \tau \ c_2 \right) \end{aligned}$$

## 5 TOOLS

In Section 3 and Section 4, we defined system specifications as well as five different fundamental problems about those specifications. In this section, we look at different formal verification tools, and for each tool we specify why our problems cannot be even expressed using these tools. All of these tools are developed solely for the purpose of model checking cyber-physical systems. Table 2 lists these tools along where they fail to support required features. We have identified six reasons. The first four prevent us from specifying our models using these tools, and the last two prevents us from specifying our verification problems using these tools. Note that we completely ignored possible difficulties in expressing our models and problems in the language of these tools, and the fact that  $C$ , finite set of cars, is given as a parameter (i.e. it is fine if a tool can solve these problems for a fixed known number of cars  $\geq 2$ ).

- **Non-Linear Dynamics:** Some tools do not support non-linear dynamics. For example, UPPAAL is for model checking timed automata, HyTech is for model checking rectangular automata, SpaceEx, PHAVer, and HARE<sub>16</sub><sup>7</sup> are for model checking hybrid automata with affine dynamics. Note that support for non-linear dynamics in HARE<sub>17</sub>, is only for flows and not discrete transitions.
- **Ordinary Differential Inclusions (ODI):** Some tools only support ordinary differential equation and not ordinary differential inclusion. In Section 3, the only constraints that we ever put on accelerations was some bound on its value in Definition 9. This means velocity is restricted using some bound on its derivative.
- **Delays in Dynamics:** None of these tools supports having delays in dynamics. In timed automata, delays in dynamics are closely related to skewed clocks, and for a very large subclass of timed automata, it is known how to handle skew clocks using UPPAAL [2, 6, 19, 23]. However, timed automata are far from what we need to express our dynamics. Note that even if we set response time ( $\rho$ ) to 0, blame time and hence policy still depend on continuous state of the system in the past.
- **Unbounded State Space:** Most tools that handle non-linear dynamics, require state space to be bounded using intervals for every state variable. However, no state variable is bounded in this paper.
- **Unbounded Time:** Similar to unbounded state space, most tools that handle non-linear dynamics require time horizon to be bounded. Note that bounding time does not necessarily bound number of discrete transitions that can be taken within the given bound [3, 24], and tools like dReach, C2E2, and FLOW\* also require number of discrete transitions to be bounded as well.
- **Robustness:** None of these tools supports specifying robustness. Similar to delays, in timed automata, robustness (as defined in this paper) is similar to perturbing constraints. If we consider perturbation of constraints for robustness, the problem has been already solved for timed automata using UPPAAL [2, 6, 19, 23]. However, not only timed automaton, is far from what we need in our specification, it is not even clear that robustness as defined here (taken from [13]) is equivalent to robustness based on perturbation of constraints.

<sup>7</sup>HARE<sub>16</sub> [21] and HARE<sub>17</sub> [22] are two different versions of the same tool. We decided to separate them since only the older version supports ordinary differential inclusion.

Name		Non- Linearity	ODI	Delays	Unbounded State	Unbounded Time	Robustness
dReach	[15]		X	X	X	X	X
SpaceEx	[12]	X	X	X	X	X	X
PHAVer	[11]	X		X			X
HyTech	[14]	X		X			X
C2E2	[8]		X	X	X	X	X
Flow*	[4]		X	X	X	X	X
HARE <sub>16</sub>	[21]	X		X			X
HARE <sub>17</sub>	[22]		X	X			X
HSolver	[20]			X	X		X
UPPAAL	[16]	X	X	X			X

**Table 2: Different model checkers and why they cannot be used to solve our problems. Cross marks are where a tool lacks a required support.**

## 6 CONCLUSION

In this paper, we presented a challenge problem for formal verification tools developed or aimed to be developed for industrial cyber-physical system. We formalized main components of dynamics and policies introduced in [25] for autonomous vehicles driving on a straight road. This also helped us to find some inconsistencies with the current specifications in [25]. To the best of our knowledge, no current automatic formal verification tool can be used to even express these dynamics and problems. We hope this serves as a challenge problem for formal tools targeting automatic verification of industrial cyber-physical systems.

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